NOISE AND TRANSFER PROPERTIES OF HARMONICALLY SYNCHRONIZED OSCILLATORS

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ABSTRACT

A theoretical model of a harmonically synchronized oscillator is developed and verified experimentally. Using this model output power, stability conditions, noise and transfer properties of a harmonic frequency divider are calculated and discussed.

Introduction

Harmonic synchronization of a free-running oscillator is a powerful means for performing frequency division in the microwave region. A possible application of such a divider circuit lies e.g. in an indirect amplification system for FM signals, which consists of a frequency divider, a power amplifier in the lower GHz-region, and a final frequency multiplier.

This work is devoted to an analysis of harmonically synchronized oscillators. Such a synchronization is based on a nonlinear interaction process of synchronizing and synchronized signals in the active device. Hence a linearized theory as that of Kurokawa [1] for noise in fundamentally synchronized oscillators cannot be applied here. The nonlinearity must fully be accounted for instead.

Our theory is a phenomenological one. The shape of the nonlinearity is arbitrary: it may be single-valued or double-valued as for Gunn elements, N-shaped or S-shaped. The nonlinearity is assumed to be incorporated into a one-port network (representing a negative resistance device), although the theory can be modified to include a two-port network, likewise. The mathematical treatment of the problem consists of two parts. First the rf-carrier signals will be calculated by using the describing function method[2]. The nonlinearity can then be replaced by a periodically driven network, which allows a linear analysis for the small sideband noise or modulation signals [3].

Harmonic Synchronization

It is the aim of this contribution to develop a theory, which leads to a tractable description of the performance of harmonically synchronized oscillators. This oscillator model shall complete the linearized model of Kurokawa with respect to including inherently nonlinear effects as harmonic (or subharmonic) synchronization. It is based on a simple equivalent circuit, which is shown in fig. 1. The resonance structure is approximated by a parallel LC-circuit with load conductance G_L, N means the active device, and v_i is a synchronizing voltage source. The active device shall be described by a cubic (van-der-Pol) current-voltage characteristic, relating the

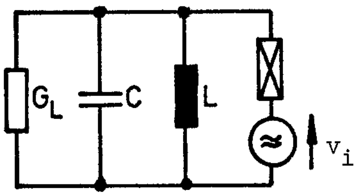

Fig.1 Equivalent circuit of harmonically synchronized oscillator

normalized device current y to the normalized device voltage x via

$$y=-x+ax^2+x^3=f(x). \qquad (1)$$

The normalizing quantities have been chosen according to [2]. 'a' is a dimensionless parameter.

The equivalent circuit of the synchronized oscillator may in fact be as general as necessary. It may e.g. even contain nonreciprocal elements. For only two assumptions have to be made for the analysis:

1. The nonlinear characteristic is known either analytically or numerically.
2. The filtering effect of the linear part of the network is such that the voltage waveform across the active device can be guessed.

The oscillation amplitude can then be calculated by using the describing function method [2]: The voltage waveform being known, the current waveform is calculated by Fourier analysis using (1). Then the fundamental components of current and voltage are related by the so-called describing function N, which means the effective admittance of the nonlinear device. By setting

$$x=\hat{x}_1 \cos(\omega t+\varphi) + \hat{x}_i \cos(m\omega t) \qquad (2)$$

with m an integer, the normalized effective admittance n can be calculated. It depends on $\hat{x}_1$, $\hat{x}_i$, and φ for m=2 and m=3. Synchronization [1] with higher harmonics cannot be described by a cubic nonlinearity, because n turns out to be purely real in these cases. This means, that the oscillation frequency cannot be tuned away from the natural frequency of the LC-circuit: synchronization is not possible. In order to treat synchronization with $m>3$, higher order terms in the characteristic (1) have to be retained.

The present choice of a cubic nonlinearity allows an explicit calculation of the oscillation amplitude and phase. Furthermore, group delay distortion can be derived by differentiating the phase with respect to the frequency.

Stability Conditions

A disadvantage of the describing function method is, that it does not allow to determine the stable synchronization range [1]. We will hence describe a new stability criterion, which is based on the conservation of power.

The method is based on one of the most general minimum principles of physics, the Gaussian principle of the least constraint. Although it has originally been formulated in analytical mechanics, it can be transferred to electromagnetic theory. Here the constraint Z turns out to be the first time derivative of the power P [4] :

$$Z = dP / dt. \quad (3)$$

The Gaussian principle of the least constraint then reads

$$\delta Z = \delta(dP / dt) = 0, \quad (4)$$

i.e. the first variation of the constraint vanishes.

Applying (4) to oscillatory systems yields information about their stability. To this end the oscillator and the locking source are surrounded by a boundary surface. This leads to a closed system, whose state is characterized by a complex power balance equation. Writing the total power as

$$P = P_a + jP_r - \mathrm{Re}(P_i) - j\mathrm{Im}(P_i) \quad (5)$$

with P_a the active, P_r the reactive, and P_i the injected power, the principle of conservation of energy P=0 yields the already-known voltage amplitude and phase.

The total power in (5) depends on both the voltage amplitude and the oscillation frequency. In order to evaluate (4), we regard a small perturbation of the operating point. The new state is characterized by a complex frequency [1] : $\omega=\omega_r - j\alpha$. Eq.(4) then yields 2 relations with $\delta\omega_r$, $\delta\alpha$, and $\delta\hat{x}_1$ as variables, which are combined in order to eliminate $\delta\omega_r$. A stability criterion can be gained from this relation, if one takes into account, that an increase in $\hat{x}_1$ must cause an increase in the damping term $e^{\alpha t}$. Thus the stable synchronization range can be determined as a function of P_i or $\hat{x}_i$.

Noise Performance

In order to investigate the noise performance of harmonically synchronized oscillators, sideband vectors of small magnitude and stochastic phase are introduced as perturbations of the carrier signals. The calculation closely follows the pattern first introduced into oscillator theory by Hines [5]. The noise mechanism in the active device is taken into account by equivalent sideband noise voltage sources. These sources have to be thought of as being in series to the locking voltage source v_i in fig.1. We are thus capable of describing both intrinsic and injected noise in a similar way. While the latter is superposed on the locking signal (the sidebands are hence located at $m\omega\pm\Omega$ with Ω the 'distance to carrier frequency'), the former exists not only close to the fundamental frequency but also close to its harmonics: the intrinsic noise vectors must be assumed to be located at $i\omega\pm\Omega$ with i=1,2,3... This is valid as long as the spectrum of the intrinsic noise is white.

The parametric mixing process between the various noise vectors is described by linear matrix algebra. The size of the computations is considerably reduced in the case, that the equivalent circuit of fig.1 is analyzed in conjunction with a cubic nonlinearity. Superposing small perturbation currents Δy and voltages Δx on the carrier signals y and x means for (1)

$$y+\Delta y = f(x+\Delta x), \; \Delta y \ll y, \; \Delta x \ll x. \quad (6)$$

Eq.(6) can be linearized with respect to the noise signals:

$$y = f'(x)\Delta x, \; f'(x)=df/dx=-1+2ax+3x^2, \quad (7)$$

where $f'(x)$ is a periodic function of time and may hence be expanded into a Fourier series. In the case of the cubic nonlinearity, the Fourier series is finite with coefficients $g_k=0$ for $k \geq 2m+1$. The highest pair of sidebands, which must be taken into account, is hence located at $2m\omega\pm\Omega$.

The scheme for analyzing the equivalent circuit of fig.1 for the noise signals is the following: At the nonlinear element, the noise currents at frequencies $i\omega\pm\Omega$ with i=1 to i=2m are related to the noise voltages by a matrix equation with g_k being the elements of the conversion matrix. These equations are completed by Kirchhoff's voltage law. The upper and lower sideband voltages at the fundamental frequency Δx_u and Δx_l can then be calculated by superposing the contributions from the various sideband pairs of noise voltage sources. Finally one has to average over the stochastic phases of the noise sources. The AM-noise spectrum is then given by

$$\phi_{AM} \sim \overline{\mathrm{Re}(\Delta x_u + \Delta x_l)^2} \quad (8a)$$

and the PM-noise spectrum by

$$\phi_{PM} \sim \overline{\mathrm{Im}(\Delta x_u + \Delta x_l)^2}. \quad (8b)$$

In (8) the bar denotes the averaging process.

AM to PM- and PM to AM-conversion can be calculated, when the injected noise is assumed to be either purely AM or purely PM. In the former case the pair of sideband sources is given by

$$\Delta x_{iu} \sim e^{-j\gamma}, \; \Delta x_{il} \sim e^{j\gamma} \; (\mathrm{AM}), \quad (9a)$$

in the latter by

$$\Delta x_{iu} \sim e^{-j\gamma}, \; \Delta x_{il} \sim -e^{j\gamma} \; (\mathrm{PM}). \quad (9b)$$

γ is the common stochastic phase.

Simple expressions for the noise spectra can be given for some special cases: If one takes only the intrinsic noise close to the fundamental frequency into account, the output noise is given by

$$\phi_{AM} \sim \left(\frac{g_L}{g_o+g_L+g_2}\right)^2, \; \phi_{PM} \sim \left(\frac{g_L}{g_o+g_L-g_2}\right)^2, \nu=1, \; \Omega\to 0. \quad (10)$$

Furthermore injected AM-noise leads to

$$\phi_{AM} \sim \left[\frac{(g_1+g_3)(g_2-g_o-g_L)}{(g_o+g_L)^2-g_2^2}\right]^2 \text{ for } m=2,$$

$$\emptyset_{AM} \sim \left[\frac{(g_2+g_4)(g_2-g_o-g_L)}{(g_o+g_L)^2-g_2^2}\right]^2 \quad \text{for } m=3, \quad (11)$$

and injected PM-noise to

$$\emptyset_{PM} \sim \left[\frac{(g_1-g_3)(g_2+g_o+g_L)}{(g_o+g_L)^2-g_2^2}\right]^2 \quad \text{for } m=2,$$

$$\emptyset_{PM} \sim \left[\frac{(g_2-g_4)(g_2+g_o+g_L)}{(g_o+g_L)^2-g_2^2}\right]^2 \quad \text{for } m=3. \quad (12)$$

Eqs.(11) and (12) are valid for $\nu=1$ and $\Omega \to 0$. ν means the normalized oscillation frequency.

Results

Some advantageous features of frequency division by harmonic synchronization are well-known: Such a divider behaves as a power limiter and shows power gain. Furthermore, the magnitude of the stable locking range is almost as large as that of a fundamentally synchronized oscillator in the case of m=2. For m=3 half of this value can be obtained. Comparing with fundamental synchronization one can state as a rule of thumb, that 10 dB more locking power is needed, in order to attain equal locking ranges.

A characteristic feature of harmonic synchronization is that the locking range reaches a maximum versus the injection power. This is due to the conversion efficiency, which depends on the injected signal. A large drive of the nonlinear device reduces that portion of the injected signal, which is downconverted to the fundamental. Hence the effective synchronizing signal decreases as does the locking range.

The maximum locking range has been computed to amount to 13 per cent for m=2, 0 dB power gain and -1 dB output power compression, and to 4 per cent for m=3, 6 dB power gain and 0 dB output power compression. The loaded Q-factor Q_L had been set to $Q_L=10$. (By the way: the dependence of the locking range on the loaded Q-factor is the same for fundamental and harmonic synchronization.) One can nevertheless state, that a gain of 10 dB should be possible. In this case the locking range amounts to 6 per cent for m=2 and to 3 per cent for m=3 ($Q_L=10$).

The phase shift across the locking band behaves similar as for fundamental synchronization:

$$\tan(m\varphi) = K\cdot(\nu - 1/\nu). \quad (13)$$

K is a function of the injection signal $\hat{x}_i$. Group delay is hence smaller by a factor of 1/m. The maximum phase shift at the borders of the locking band is $\pi/(2m)$.

The dependence of the intrinsic noise on the injected power will be discussed next. As for fundamental synchronization the PM-noise is drastically reduced, if a stable harmonic signal is injected. This is shown for m=3 in fig.2. The normalized PM sideband noise power P_{PM} has been drawn versus X_i, which is proportional to the injected power. Increasing X_i yields decreasing PM-noise, until a minimum has been reached at a relatively large X_i. If X_i is increased beyond this point, the PM-noise becomes worse. The explanation is the same as for the existence of a maximum of the locking range: the stabilizing effect of the injection signal depends on the conversion efficiency and hence on the drive of the nonlinear element.

It can further be seen from fig.2, that the noise components close to the harmonics contribute about as much to the PM-noise as do the fundamental components alone. (In cases b and d, noise sources at ω and iω with i=2 to i=2m have been taken into account.)

The output PM-noise due to the PM-noise of the injection signal is shown versus X_i in fig.3. This relationship is completely different from the corresponding one for fundamental synchronization, where the PM-noise at the output does not depend on the injection power[6]. For harmonic synchronization, however, the PM-noise monotonically decreases versus X_i. Again lies the explanation for this effect in the conversion efficiency, which decreases versus X_i. Hence the PM-noise which is downconverted from around mω to ω must decrease, too. Combining the results of figs. 2 and 3 one must suppose, that the total PM-noise at the output should have a minimum versus X_i. This effect could indeed be observed in practice.

An unexpected result was that the PM-noise is less for $\nu \neq 1$ than for $\nu=1$, i.e. it shows a maximum in the middle of the stable locking range (look at case b in fig.3!). This is due to a very strong PM to AM conversion, which occurs for $\nu \neq 1$ at locking powers less than -10 dB. Such an enhancement in AM-noise may impair the overall performance of the divider in practice.

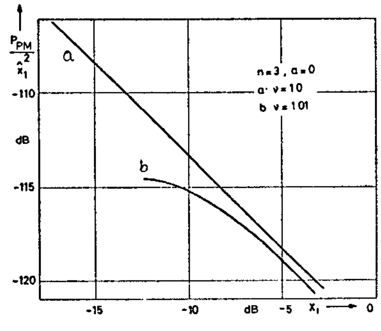

Fig.3 PM-noise due to PM-injection

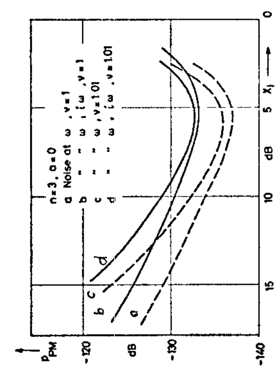

Fig.2 Intrinsic PM

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